

26/10/2016 الثلاثاء

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محاضرة [5]

* Common methods of defuzzification:

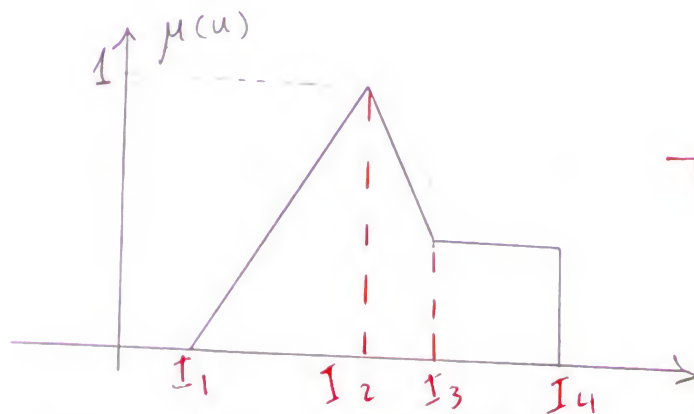
① Center of Gravity Method (COG)

≡ Center of area method (COA)

≡ The centroid method

$$u^{crisp} = \frac{\int \mu(u) \cdot u \, du}{\int \mu(u)}$$

Example:



The overall fuzzy
o/p

$$u^{crisp} = \frac{\int \mu(u) \cdot u \, du}{\int \mu(u)} = \frac{\int_{I_1}^{I_2} \mu(u) \cdot u \, du + \int_{I_2}^{I_3} \mu(u) \cdot u \, du + \int_{I_3}^{I_4} \mu(u) \cdot u \, du}{\int_{I_1}^{I_2} \mu(u) \, du + \int_{I_2}^{I_3} \mu(u) \, du + \int_{I_3}^{I_4} \mu(u) \, du}$$

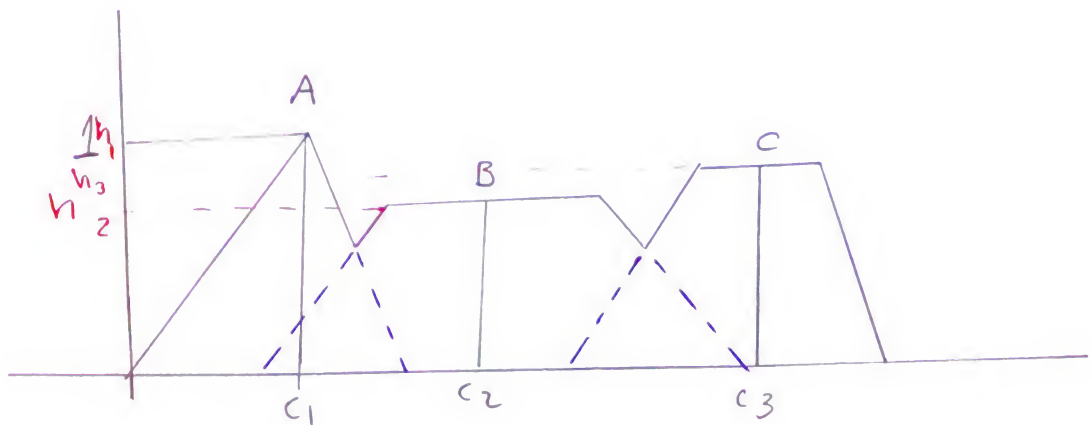
THE MOST ACCURATE METHOD

if eqns are hard to obtain, we use discrete method

$$\mu^{crisp} = \frac{\sum_{i=1}^n \mu(u_i) \cdot u_i}{\sum_{i=1}^n \mu(u_i)} \quad \leftarrow \text{discrete (COA)}$$

[2] Max-Mean Membership method:

Example



$$U^{crisp} = \frac{c_1 + c_2 + c_3}{3}$$

$$\Rightarrow U^{crisp} = \frac{\sum_{k=1}^N c_k}{N}$$

c_k is the corresponding value to peaks of o/p fuzzy sets.

[3] Weighted Average Method: المتوسط المرجح

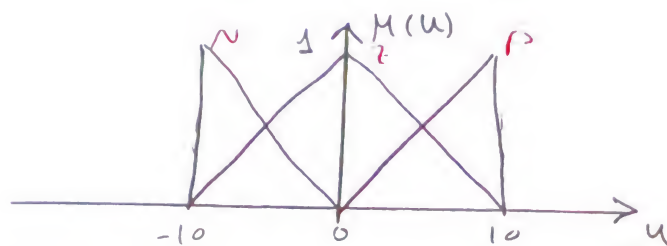
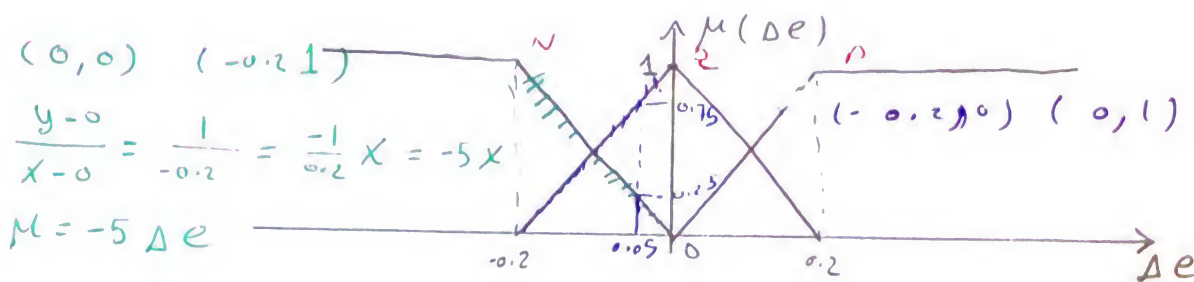
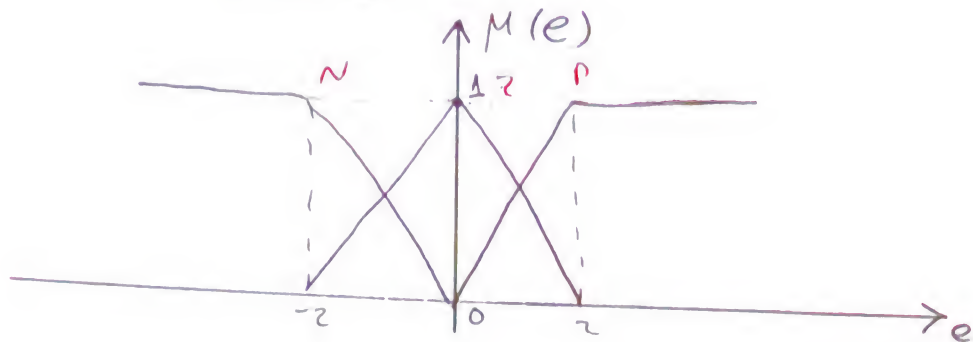
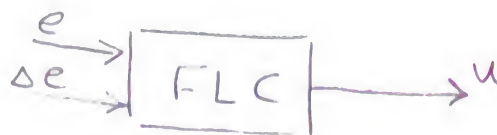
$$U^{crisp} = \frac{\sum_{i=1}^N \mu(u_i) \cdot u_i}{\sum_{i=1}^N \mu(u_i)}$$

where u_i is the symmetric point to the fuzzy set o/p form.

for the previous figure

$$U^{crisp} = \frac{c_1 h_1 + c_2 h_2 + c_3 h_3}{h_1 + h_2 + h_3}$$

Example



Find the crisp output in the following cases

- ① $e = -2$ & $\Delta e = -0.05$
- ② $e = 1.5$ & $\Delta e = -0.15$
- ③ $e = 2$ & $\Delta e = 0.2$
- ④ $e = 1$ & $\Delta e = 0.1$

Fired rules

$e \backslash \Delta e$	N	Z	P
N	P	P	Z
Z	P	Z	N
P	Z	N	N

① $e = -2$ & $\Delta e = -0.05$

$e = -2 \rightarrow N$ with $\mu = 1$ $\rightarrow e$ is N
 $\Delta e = -0.05 \rightarrow N$ with $\mu = 0.25$
 $\Delta e = -0.05 \rightarrow Z$ with $\mu = 0.75$

← μ العزم

← μ العزم

Δe is N

Δe is Z

Premise terms

[2] the fired rules (Which rules are on)

check the previous table

R_1 : if e is N and Δe is N then u is P

R_2 : if e is N and Δe is Z then u is P

estimated terms

[3] strength of fired rules

$$\begin{aligned}\mu_{P_1} &= \min \{ \mu_N(e=-2), \mu_N(\Delta e = -0.05) \} \\ &= \min \{ 1, 0.25 \} = 0.25\end{aligned}$$

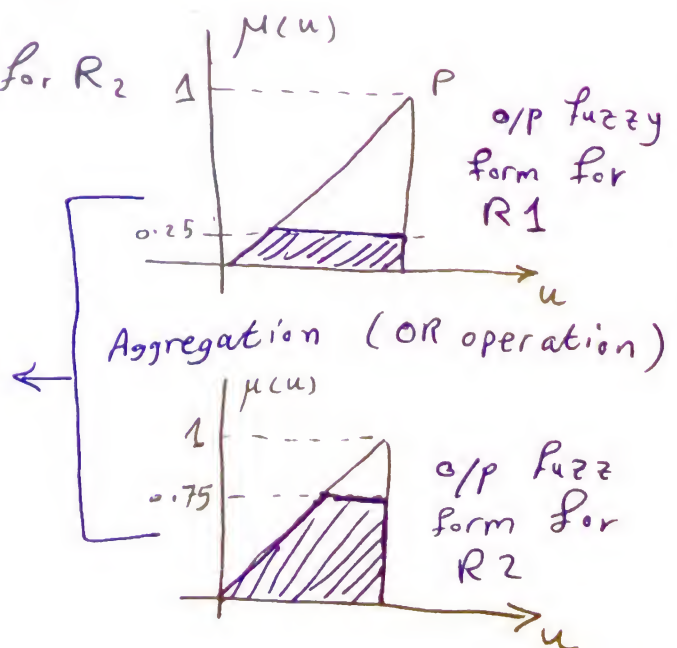
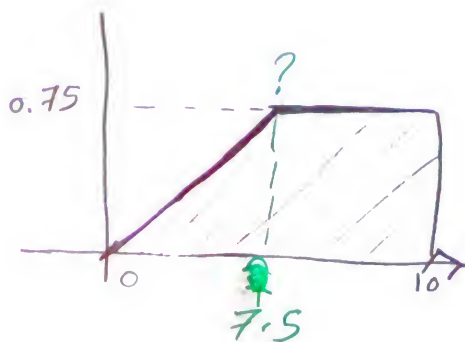
$$\begin{aligned}\mu_{P_2} &= \min \{ \mu_N(e=-2), \mu_Z(\Delta e = -0.05) \} \\ &= \min \{ 1, 0.75 \} = 0.75\end{aligned}$$

[4] The fuzzy sets output forms

$$* \mu_P(u) = \min \{ \mu_P(u), 0.25 \} \rightarrow \text{for } R_1$$

$$* \mu_P(u) = \min \{ \mu_P(u), 0.75 \} \rightarrow \text{for } R_2$$

[5] overall output of fuzzy set



⑥ defuzzification using (COG) method :-

line eqn = $0.1u$

$\rightarrow 0.75 = 0.1u \Rightarrow u = 7.5$

$$u^{\text{crisp}} = \frac{\int_0^{7.5} \mu(u) \cdot u \, du + \int_{7.5}^{10} \mu(u) \cdot u \, du}{\int_0^{7.5} \mu(u) \, du + \int_{7.5}^{10} \mu(u) \, du} = \frac{I_1}{I_2}$$

$$I_1 = \int_0^{7.5} (0.1u) u \, du + \int_{7.5}^{10} 0.75 u \, du = \left[\frac{0.1 u^3}{3} \right]_0^{7.5} + \left[0.75 \frac{u^2}{2} \right]_{7.5}^{10} = 30.468$$

$$I_2 = \int_0^{7.5} 0.1u \, du + \int_{7.5}^{10} 0.75 \, du = \left[\frac{0.1 u^2}{2} \right]_0^{7.5} + [0.75u]_{7.5}^{10} = 4.6875$$

$$u^{\text{crisp}} = \frac{I_1}{I_2} = 6.5$$

② $e = 1.5$ & $\Delta e = -0.15$

① Fuzzification :

$e = 1.5 \rightarrow \begin{cases} \rightarrow Z \text{ with } \mu = 0.25 \rightarrow e \text{ is } Z \\ \rightarrow P \text{ with } \mu = 0.75 \rightarrow e \text{ is } P \end{cases}$

$\Delta e = -0.15 \rightarrow \begin{cases} \rightarrow N \text{ with } \mu = 0.75 \rightarrow \Delta e \text{ is } N \\ \rightarrow Z \text{ with } \mu = 0.25 \rightarrow \Delta e \text{ is } Z \end{cases}$

[2] Fired ~~rules~~ rules

R_1 : if e is Z and Δe is N then u is P

R_2 : if e is P and Δe is N then u is Z

R_3 : if e is Z and Δe is Z then u is Z

R_4 : if e is P and Δe is Z then u is P

[3] strength of fired rules

$$\mu_{P_1} = \min \{ 0.25, 0.75 \} = 0.25$$

$$\mu_{P_2} = \min \{ 0.75, 0.75 \} = 0.75$$

$$\mu_{P_3} = \min \{ 0.25, 0.25 \} = 0.25$$

$$\mu_{P_4} = \min \{ 0.75, 0.75 \} = 0.75$$

[4] forms of fuzzy sets o/p

$$\mu_P(u) = \min \{ \mu_{P_1}(u), 0.25 \} \rightarrow R_1$$

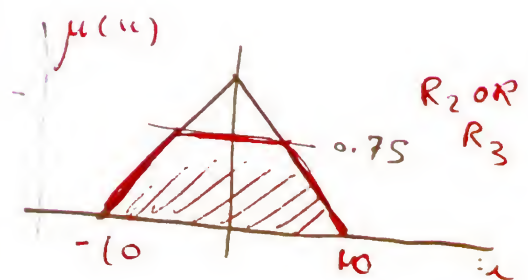
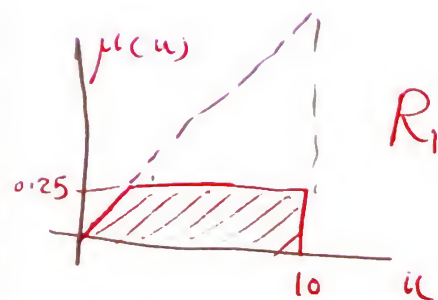
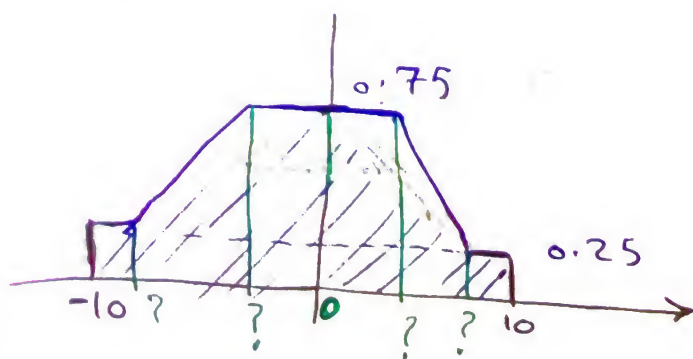
$$\mu_Z(u) = \min \{ \mu_{P_2}(u), 0.75 \} \rightarrow R_2$$

$$\mu_Z(u) = \min \{ \mu_{P_3}(u), 0.25 \} \rightarrow R_3$$

$$\mu_N(u) = \min \{ \mu_{P_4}(u), 0.75 \} \rightarrow R_4$$

[5] overall fuzzy output

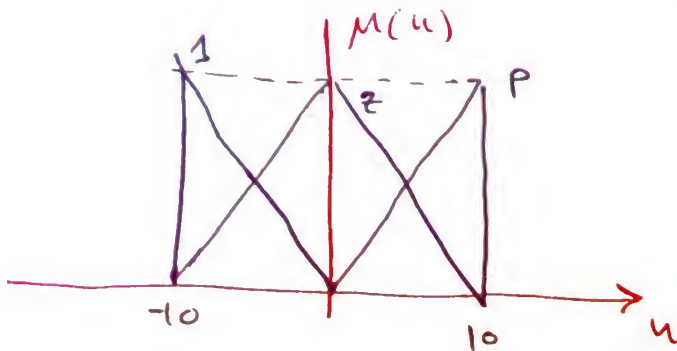
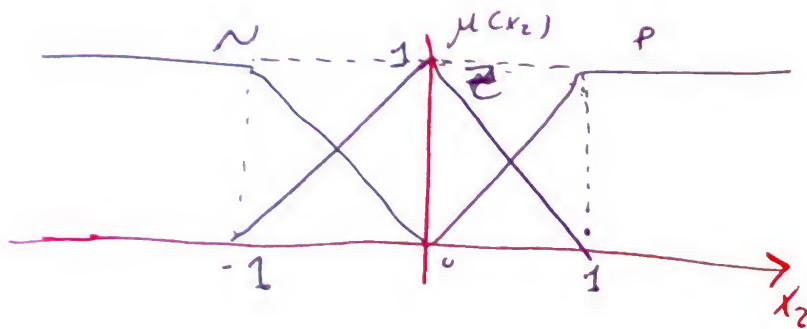
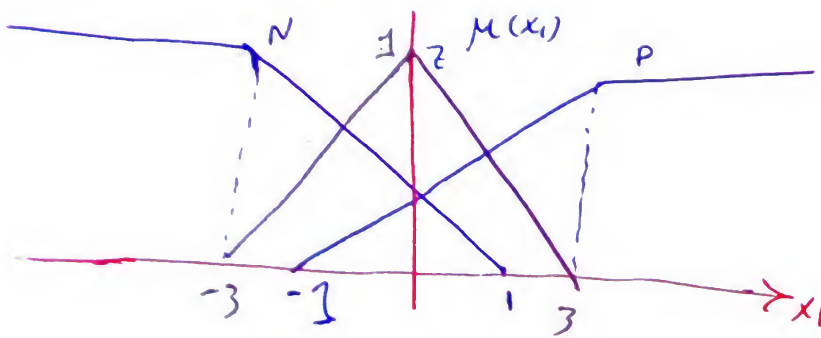
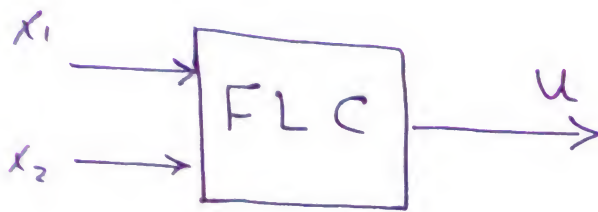
Aggregation (OR operation)



* Because the shape is symmetrical

$$U^{\text{crisp}} = 0$$

REPORT



Find the
Crisp o/p
if $x_1 = 0.5$;
 $x_2 = 0$
For same
table of rules
in previous
example
using COG
method